Big-bang nucleosynthesis with a long-lived CHAMP including He4 spallation process

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Abstract. We propose helium-4 spallation processes induced by long-lived stau in supersymmetric standard models, and investigate an impact of the processes on light elements abundances. We show that, as long as the phase space of helium-4 spallation processes is open, they are more important than stau-catalyzed fusion and hence constrain the stau property. This talk is based on works [1].

1. Introduction

Long-lived charged massive particles (CHAMPs) will play interesting roles in the Big-Bang Nucleosynthesis (BBN). The light nuclei will interact not only with the CHAMPs during the BBN processes [2, 3, 4, 5, 6, 7], but also with the decay products of the CHAMPs in the post-BBN era [8, 9]. The standard BBN will thus be altered, and so is the abundance of the light elements at the present time. One can thus constrain the models beyond the Standard Model by evaluating their prediction on the light elements abundance and comparing it with the current observations. We can then give stringent predictions for the forthcoming experiments and observations according to these constraints.

The Standard Model extended with supersymmetry (SUSY) is one of the models that can accommodate such long-lived CHAMPs. With the R-parity conservation, the lightest SUSY particle (LSP) is stable and become a cold dark matter. Interestingly, it can offer a long-lived CHAMPs if the LSP is the bino-like neutralino $\tilde{\chi}_1^0$. Coannihilation mechanism is required to account for the dark matter abundance in this case [10], where the LSP and the next-lightest SUSY particle (NLSP) are almost degenerate in mass. Staus, denoted by $\tilde{\tau}$ and a possible candidate of the NLSP, can acquire a long lifetime when the mass difference with the LSP is less than the mass of tau leptons. This is due to the phase space suppression of the final state that necessarily consists of three particles or more. Noting that such long-lived staus will be copious during the BBN [11, 12], we have shown in [3, 5, 7] that their presence indeed alters the prediction of the standard BBN and possibly solve the discrepancy of the lithium abundance in the Universe through the internal conversion reactions.

In this talk, we propose new reactions

$$(\tilde{\tau}\,{}^4\mathrm{He}) \to \tilde{\chi}^0_1 + \nu_\tau + \mathrm{t} + \mathrm{n}, \tag{1a}$$

$$(\tilde{\tau}\,{}^4\mathrm{He}) \to \tilde{\chi}^0_1 + \nu_\tau + \mathrm{d} + \mathrm{n} + \mathrm{n}, \tag{1b}$$

$$(\tilde{\tau}\,{}^4\mathrm{He}) \to \tilde{\chi}^0_1 + \nu_\tau + \mathrm{p} + \mathrm{n} + \mathrm{n} + \mathrm{n}, \tag{1c}$$

in which $(\tilde{\tau}\,{}^4\mathrm{He})$ represents a bound state of a stau and ^{4}He nucleus. Reaction (1) is essentially a spallation of the ^{4}He nucleus, producing a triton t, a deuteron d, and neutrons n. Presence of such spallation processes has been ignored so far due to the naïve expectation that the rate of the stau-catalyzed fusion [2]

$$(\tilde{\tau}\,{}^4\mathrm{He}) + \mathrm{d} \to \tilde{\tau} + {}^6\mathrm{Li} \tag{2}$$

is larger than the reaction (1). Indeed, the cross section of Eq. (2) is much larger than that of ${}^4\mathrm{He} + \mathrm{d} \to {}^6\mathrm{Li} + \gamma$ by $(6-7)$ orders of magnitude [13].

We point out that this expectation is indeed naïve; the reaction Eq. (1) is more effective than Eq. (2) as long as the spallation processes are kinematically allowed. The former reaction rapidly occurs due to the large overlap of their wave functions in a bound state. On the other hand, the latter proceeds slowly since it requires an external deuteron which is sparse at the BBN era. The overproduction of t and d is more problematic than that of ^{6}Li. This puts new constraints on the parameters of the minimal supersymmetric standard model (MSSM). Note that there is no reaction corresponding to Eq. (1) in the gravitino LSP scenario [14, 15].

2. Spallation of helium 4

The ^{4}He spallation processes of Eq. (1) is described by the Lagrangian

$$\mathcal{L} = \tilde{\tau}^* \overline{\tilde{\chi}^0_1}(g_\mathrm{L} P_\mathrm{L} + g_\mathrm{R} P_\mathrm{R})\tau + \sqrt{2} G_\mathrm{F} \nu_\tau \gamma^\mu P_\mathrm{L} \tau J_\mu + \mathrm{h.c.}, \tag{3}$$

where $G_\mathrm{F} = 1.166 \times 10^{-5}\mathrm{GeV}^{-2}$ is the Fermi coupling constant, $P_{\mathrm{L(R)}}$ represents the chiral projection operator, and J_μ is the weak current. The effective coupling constants g_L and g_R are given by

$$g_\mathrm{L} = \frac{g}{\sqrt{2}\cos\theta_\mathrm{W}} \sin\theta_\mathrm{W}\cos\theta_\tau, \;\; g_\mathrm{R} = \frac{\sqrt{2}g}{\cos\theta_\mathrm{W}} \sin\theta_\mathrm{W}\sin\theta_\tau \mathrm{e}^{i\gamma_\tau}, \tag{4}$$

where g is the $SU(2)_\mathrm{L}$ gauge coupling constant and θ_W is the Weinberg angle. The mass eigenstate of staus is given by the linear combination of $\tilde{\tau}_\mathrm{L}$ and $\tilde{\tau}_\mathrm{R}$, the superpartners of left-handed and right-handed tau leptons, as

$$\tilde{\tau} = \cos\theta_\tau \tilde{\tau}_\mathrm{L} + \sin\theta_\tau \mathrm{e}^{-i\gamma_\tau} \tilde{\tau}_\mathrm{R}. \tag{5}$$

Here θ_τ is the left-right mixing angle of staus and γ_τ is the CP violating phase.

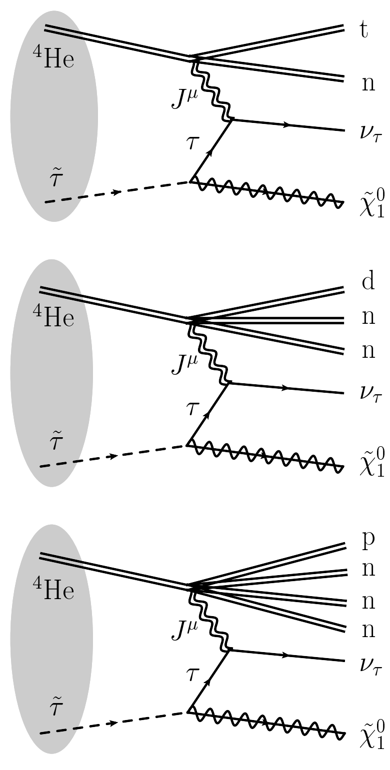

Figure 1. ^{4}He spallation processes.

2.1. $(\tilde{\tau}\,^4He) \to \tilde{\chi}^0_1 + \nu_\tau + t + n$

First we consider the process of Eq. (1a). The rate of this process is expressed as

$$\frac{1}{\tau_{\rm tn}} = \frac{1}{|\psi|^2 \cdot \sigma v_{\rm tn}}, \tag{6}$$

where $|\psi|^2$ stands for the overlap of the wave functions of the stau and the ^{4}He nucleus. We estimate the overlap by

$$|\psi|^2 = \frac{(Z\alpha m_{\rm He})^3}{\pi}, \tag{7}$$

where Z and $m_{\rm He}$ represent the atomic number and the mass of ^{4}He, respectively, and α is the fine structure constant. We assumed that the stau is pointlike particle and is much heavier than ^{4}He nucleus so that the reduced mass of the bound state is equal to the mass of ^{4}He nucleus itself. The cross section of the elementary process for this reaction is denoted by $\sigma v_{\rm tn}$ and calculated as

$$\begin{aligned} \sigma v_{\rm tn} &\equiv \sigma v\big((\tilde{\tau}^4{\rm He}) \to \tilde{\chi}^0_1 \nu_\tau {\rm tn}\big) \\ &= \frac{1}{2E_{\tilde{\tau}}} \int \frac{d^3\boldsymbol{p}_\nu}{(2\pi)^3 2E_\nu} \frac{d^3\boldsymbol{p}_{\tilde{\chi}}}{(2\pi)^3 2E_{\tilde{\chi}}} \frac{d^3\boldsymbol{q}_{\rm n}}{(2\pi)^3} \frac{d^3\boldsymbol{q}_{\rm t}}{(2\pi)^3} \\ &\qquad \times \big|\mathcal{M}\big((\tilde{\tau}^4{\rm He}) \to \tilde{\chi}^0_1 \nu_\tau {\rm tn}\big)\big|^2 \\ &\qquad \times (2\pi)^4 \delta^{(4)}(p_{\tilde{\tau}} + p_{\rm He} - p_\nu - q_{\rm t} - q_{\rm n}). \end{aligned} \tag{8}$$

Here p_i and E_i are the momentum and the energy of the particle species i, respectively.

We briefly show the calculation of the amplitude of this process. The amplitude is deconstructed as

$$\begin{aligned} \mathcal{M}\big((\tilde{\tau}^4{\rm He}) \to \tilde{\chi}^0_1 \nu_\tau {\rm tn}\big) &= \langle {\rm t\,n}\, \tilde{\chi}^0_1\, \nu_\tau | \mathcal{L}_{\rm int} |^4{\rm He}\, \tilde{\tau}\rangle \\ &= \langle {\rm t\,n} | J^\mu |^4{\rm He}\rangle \, \langle \tilde{\chi}^0_1\, \nu_\tau | j_\mu | \tilde{\tau}\rangle. \end{aligned} \tag{9}$$

The weak current J_μ consists of a vector current V_μ and an axial vector current A_μ as $J_\mu = V_\mu + g_{\rm A} A_\mu$, where $g_{\rm A}$ is the axial coupling constant. The relevant components of the currents in this reaction are V^0 and A^i ($i = 1, 2, 3$). We take these operators as a sum of a single-nucleon operators as

$$V^0 = \sum_{a=1}^{4} \tau_a^- {\rm e}^{{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}_a}, \quad A^i = \sum_{a=1}^{4} \tau_a^- \sigma_a^i {\rm e}^{{\rm i}\boldsymbol{q}\cdot\boldsymbol{r}_a}, \tag{10}$$

where $\boldsymbol{q}$ is the momentum carried by the current, $\boldsymbol{r}_a$ is the spatial coordinate of the a-th nucleon ($a \in \{1, 2, 3, 4\}$), and τ_a^- and σ_a^i denote the isospin ladder operator and the spin operator of the a-th nucleon, respectively. Each component leads to a part of hadronic matrix element:

$$\begin{aligned} \langle {\rm tn} | V^0 |^4{\rm He}\rangle &= \sqrt{2}\mathcal{M}_{\rm tn}, \\ \langle {\rm tn} | g_{\rm A} A^+ |^4{\rm He}\rangle &= \sqrt{2} g_{\rm A}\mathcal{M}_{\rm tn}, \\ \langle {\rm tn} | g_{\rm A} A^- |^4{\rm He}\rangle &= -\sqrt{2} g_{\rm A}\mathcal{M}_{\rm tn}, \\ \langle {\rm tn} | g_{\rm A} A^3 |^4{\rm He}\rangle &= -\sqrt{2} g_{\rm A}\mathcal{M}_{\rm tn}, \end{aligned} \tag{11}$$

Table 1. Input values of the matter radius $R_{\rm mat}$ for d, t, and ^{4}He, the magnetic radius $R_{\rm mag}$ for p and n, nucleus mass m_X, excess energy Δ_X for the nucleus X, and each reference.

nucleus	$R_{\rm mat(mag)}$ [fm]/[GeV^{-1}]	m_X [GeV]	Δ_X [GeV]
p	0.876 / 4.439 [16]	0.9383 [21]	6.778×10^{-3} [22]
n	0.873 / 4.424 [17]	0.9396 [21]	8.071×10^{-3} [22]
d	1.966 / 9.962 [18]	1.876 [22]	1.314×10^{-2} [22]
t	1.928 / 9.770 [19]	2.809 [22]	1.495×10^{-2} [22]
^{4}He	1.49 / 7.55 [20]	3.728 [22]	2.425×10^{-3} [22]

where $A^{\pm} = (A^1 \pm iA^2)/\sqrt{2}$. Given the relevant wave functions of a ^{4}He nucleus, a triton, and a neutron (see Appendix in Ref. [1]), we obtain the hadronic matrix element as

$$\mathcal{M}_{\rm tn} = \left(\frac{128\pi}{3}\frac{a_{\rm He}a_{\rm t}^2}{(a_{\rm He}+a_{\rm t})^4}\right)^{3/4}\left\{\exp\left[-\frac{\boldsymbol{q}_{\rm t}^2}{3a_{\rm He}}\right]-\exp\left[-\frac{\boldsymbol{q}_{\rm n}^2}{3a_{\rm He}}-\frac{(\boldsymbol{q}_{\rm t}+\boldsymbol{q}_{\rm n})^2}{6(a_{\rm He}+a_{\rm t})}\right]\right\}. \tag{12}$$

Here $\boldsymbol{q}_{\rm t}$ and $\boldsymbol{q}_{\rm n}$ are three-momenta of the triton and the neutron, respectively, and $a_{\rm He}$ and $a_{\rm t}$ are related to the mean square matter radius $R_{\rm m}$ by

$$a_{\rm He} = \frac{9}{16}\frac{1}{(R_{\rm m})^2_{\rm He}},\ \ a_{\rm t} = \frac{1}{2}\frac{1}{(R_{\rm m})^2_{\rm t}}. \tag{13}$$

We list in Table 1 input values of the matter radius for the numerical calculation in this article.

The remaining part is straightforwardly calculated to be

$$\begin{aligned}|\langle\tilde{\chi}_1^0\nu_\tau|j_0|\tilde{\tau}\rangle|^2 &= |\langle\tilde{\chi}_1^0\nu_\tau|j_z|\tilde{\tau}\rangle|^2 = 4G_{\rm F}^2|g_{\rm R}|^2\frac{m_{\tilde{\chi}_1^0}E_\nu}{m_\tau^2},\\ |\langle\tilde{\chi}_1^0\nu_\tau|j_\pm|\tilde{\tau}\rangle|^2 &= 4G_{\rm F}^2|g_{\rm R}|^2\frac{m_{\tilde{\chi}_1^0}E_\nu}{m_\tau^2}\left(1\mp\frac{p_\nu^z}{E_\nu}\right),\end{aligned} \tag{14}$$

where E_ν and p_ν^z are the energy and the z-component of the momentum of the tau neutrino, respectively. We assumed that the stau and the neutralino are non-relativistic. This equation includes not only all the couplings such as $G_{\rm F}$, $g_{\rm L}$, and $g_{\rm R}$, but also the effect of the virtual tau propagation in the Fig. 1. Note here that $g_{\rm L}$ coupling does not contribute. This is because the virtual tau ought to be left-handed at the weak current, and it flips its chirality during the propagation since the transferred momentum is much less than its mass.

Combining hadronic part with the other part, we obtain the squared amplitude as

$$\left|\mathcal{M}\big((\tilde{\tau}^4{\rm He})\to\tilde{\chi}_1^0\nu_\tau{\rm tn}\big)\right|^2 = \frac{8m_{\tilde{\chi}_1^0}G_{\rm F}^2|g_{\rm R}|^2}{m_\tau^2}(1+3g_A^2)E_\nu|\mathcal{M}_{\rm tn}|^2. \tag{15}$$

Integrating on the phase space of the final states, we obtain the cross section as

$$\sigma v_{\rm tn} = \frac{8}{\pi^2}\left(\frac{32}{3\pi}\right)^{3/2}g^2\tan^2\theta_W\sin^2\theta_\tau(1+3g_A^2)G_F^2\Delta_{\rm tn}^4\ \frac{m_{\rm t}m_{\rm n}}{m_{\tilde{\tau}}m_\tau^2}\ \frac{a_{\rm He}^{3/2}a_{\rm t}^3}{(a_{\rm He}+a_{\rm t})^5}\ I_{\rm tn}, \tag{16}$$

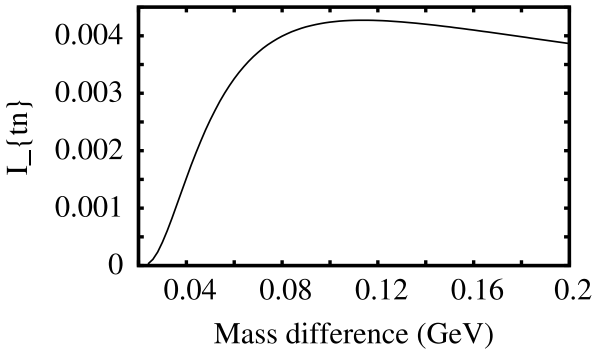

Figure 2. Factor I_{tn} in Eq. (16) as a function of mass difference between the stau and the neutralino. Here we took $m_{\tilde{\tau}} = 350\mathrm{GeV}$, $\sin\theta_\tau = 0.8$, and $\gamma_\tau = 0$.

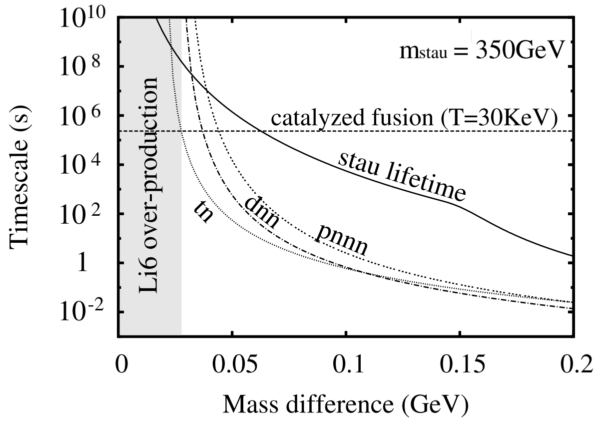

Figure 3. Timescale of spallation processes as a function of δm and the stau-catalyzed fusion at the universe temperature $T = 30\mathrm{keV}$ [13]. The lifetime of free $\tilde{\tau}$ (solid line) is also depicted. Here we took $m_{\tilde{\tau}} = 350\mathrm{GeV}$, $\sin\theta_\tau = 0.8$, and $\gamma_\tau = 0$.

Here I_{tn} is the numerically calculated factor including the information of phase space for this process. Analytic form of I_{tn} is shown in Ref. [1], and numerical result is depicted in Fig. 2. Δ_{tn}, k_{t}, and k_{n} are defined as

$$\begin{aligned} &\Delta_{\mathrm{tn}} \equiv \delta m + \Delta_{\mathrm{He}} - \Delta_{\mathrm{t}} - \Delta_{\mathrm{n}} - E_{\mathrm{b}}, \\ &k_{\mathrm{t}} \equiv \sqrt{2m_{\mathrm{t}}\Delta_{\mathrm{tn}}}\ , \quad k_{\mathrm{n}} \equiv \sqrt{2m_{\mathrm{n}}\Delta_{\mathrm{tn}}}\ , \end{aligned} \tag{17}$$

where Δ_X is the excess energy of the nucleus X, and E_{b} is the binding energy of $(\tilde{\tau}\,{}^4\mathrm{He})$ system.

3. Comparing the rate of spallation reaction with that of stau-catalyzed fusion

We compare the rate of the spallation and that of the stau-catalyzed fusion. We first note that the rate of stau-catalyzed fusion strongly depends on the temperature [13], and we fix the reference temperature to be 30keV. Staus begin to form a bound state with ^{4}He at this temperature, which corresponds to cosmic time of 10^3s. Thus the bound state is formed when the lifetime of staus is longer than 10^3s.

Figure 3 shows the timescale of the spallation processes as a function of δm. The lifetime of free stau is plotted by a solid line. We took the reference values of $m_{\tilde{\tau}} = 350\mathrm{GeV}$, $\sin\theta_\tau = 0.8$, and $\gamma_\tau = 0$. The inverted rate of the stau-catalyzed fusion at the temperature of 30keV is also shown by the horizontal dashed line. Once a bound state is formed, as long as the phase space of spallation processes are open sufficiently that is $\delta m \gtrsim 0.026\mathrm{GeV}$, those processes dominate over other processes. There $\tilde{\tau}$ property is constrained to evade the over-production of d and/or t. For $\delta m \lesssim 0.026\mathrm{GeV}$, the dominant process of $(\tilde{\tau}\,{}^4\mathrm{He})$ is stau-catalyzed fusion, since the free $\tilde{\tau}$ lifetime is longer than the timescale of stau-catalyzed fusion. Thus light gray region is forbidden due to the over-production of ^{6}Li.

This interpretation of Fig.2 is not much altered by varying the parameters relevant with $\tilde{\tau}$. First cross sections of spallation processes are inversely proportional to $m_{\tilde{\tau}}$, and then the timescale of each process linearly increases as $m_{\tilde{\tau}}$ increases. Thus, even when $m_{\tilde{\tau}}$ is larger than $m_{\tilde{\tau}} = 350\mathrm{GeV}$ by up to a factor of ten, the region of ^{6}Li over-production scarcely changes. Next we point out that our result depend only mildly on the left-right mixing of the stau. Indeed,

cross section of the ^{4}He spallation is proportional to $\sin^2\theta_\tau$. Its order of magnitude will not change as long as the right-handed component is significant.

4. Summary

Long-lived charged massive particles provides some exotic nuclear reactions in the big bang nucleosynthesis. So it is important for understanding the property of long-lived charged massive particles to understand what type of exotic nuclear induce over-production (-destruction). Newly included in the present work is the spallation of the ^{4}He in the stau-^{4}He bound state given in Eq.(1). This process is only present in the model which predicts the long-lived charged particles due to the phase space suppression with the weakly interacting daughter particle.

We calculated the rate of the helium-4 spallation processes analytically, and compared it with that of catalyzed fusion. We found that the spallation of ^{4}He nuclei dominate over the catalyzed fusion as long as the phase space of the spallation processes are open and hence the property of long lived stau is constrained from avoiding the overproduction of a deuteron and/or a triton.

Acknowledgments

This work was supported in part by the Grant-in-Aid for the Ministry of Education, Culture, Sports, Science, and Technology, Government of Japan, No. 21111006, No. 22244030, No. 23540327(K.K.), No. 24340044 (J.S.), and No. 23740208 (M.Y.).

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